Summary

Cauchy's problem is considered for the ordinary differential equation

$$\dot{x}(t) = f(t, x(t), u(t), u(t-\theta), v(t), \int_{-\sigma}^{-\tau} v(t+s))ds),\ t \in [t_0, t_1],\ \ x(t_0) = x_0, \tag{1}$$

containing discrete $u(t-\theta)$ and distribute $\int_{-\sigma}^{-\tau} v(t+s)ds$ delays in controls, which is obtained from the problem

$$\dot{x}(t) = f(t, x(t), u_0(t), u_0(t-\theta), v_0(t), \int_{-\sigma}^{-\tau} v_0(t+s))ds),\ x(t_0) = x_{00} \tag{2}$$

by perturbations of the control $u_0(t)$, $v_0(t)$ functions and the initial vector x_{00}.
The analytic representation formula is proved

$$x(t) = x_0(t) + \delta x(t;\delta w) + o(t;\delta w) \tag{3}$$

for solution $x(t)$ of the problem (1), where $x_0(t)$ is solution of the problem (2), $\delta x(t;\delta w)$ is the linear operator with respect to perturbation $\delta w = (x_0 - x_{00}, u(\cdot) - u_0(\cdot),\ \ v(\cdot) - v_0(\cdot))$ and $o(t;\delta w)$ is infinitely small of the high order comparatively with perturbation δw. The linear operator $\delta x(t;\delta w)$ is constructed in the explicit form.

The novelty in the work is the formula (3) and a form of operator $\delta x(t;\delta w)$ for the equation, where taken into account distribute delay in controls. Moreover, the form of operator $\delta x(t;\delta w)$ is concretized for the linear differential equation and the demand-supply marketing differential model. On the results obtained in the work a report was made on the conference "XXXV International Enlarged Sessions of the Seminar of I. Vekua Institute of Applied Mathematics of TSU", April 21-23, 2021, http://www.viam.science.tsu.ge/enlarged/2021/programa_geo.pdf.